PSO Algorithm Assisted by Co-Kriging and Its Application to Optimal Transposition Design of Power Transformer Winding for the Reduction of Circulating Current Loss

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A numerically more efficient and accurate co-Kriging model is developed, and incorporated into particle swarm optimization to be applied to optimal design of electromagnetic devices. The sampling points in the co-Kriging consist of a few expensive data and many cheap data to save the computational efforts while increasing modeling accuracy. The proposed algorithm is validated through an analytic example, and applied to an optimal transposition design of a power transformer to minimize its circulating current loss.

***Index Terms*— Circulating current loss, co-Kriging, ordinary Kriging, optimal design, power transformer, transposition.**

I. Introduction

OPTIMIZATION PROBLEMS in electromagnetic (EM) design are popularly tackled by using heuristic search algorithms such as particle swarm optimization (PSO), differential evolution. They provide a global optimum solution in most case. Their applications to engineering problems, however, open have difficulties because they usually requires huge computational cost related with evaluating objective function values using numerical simulation such as finite element analysis (FEA) [1]. In order to overcome the difficulties, Kriging assisted heuristic optimization algorithms, where a Kriging efficiently provides approximate objective function values in place of numerically expensive FEA, are attracting more attention [2]. In general, the prediction accuracy of a Kriging is guaranteed only when enough sampling data is provided.

In an optimal transposition design of a power transformer to reduce the circulating current loss among parallel conductors, the enough sampling data are hardly available because the loss is obtained through numerically very expensive 3-D non-linear time-stepping FEA. Furthermore, due to the very big time-constant of a power transformer, a steady state circulating current loss only can be obtained after the computation of several days using a personal computer of common speed [3]. On the other hand, axisymmetric 2-D non-linear time-stepping FEA provides only rough (inaccurate) sampling data although it reduces the computing time dramatically.

Hereinafter in this paper, the sampling data from 3-D FEA and 2-D FEA will be referred to *numerically expensive data* and *cheap data*, respectively.

According to Kriging theory, both the *expensive* and *cheap data* are not expected to provide a reasonable Kriging because of the lacks of sampling data and accuracy, respectively [4].

This paper proposes, based on ordinary Kriging, a co-Kriging model which takes most of the sampling data from *cheap data* while only some from *expensive data*. The suggested co-Kriging is validated through investigations with analytic function, and combined with PSO to be applied to the optimal transposition design of a power transformer.

II. Co-Kriging Methodology

A co-Kriging exploits the correlation between *expensive* and *cheap data* to enhance the prediction accuracy based on the assumption that the *expensive* ones are always more accurate than the *cheap* ones [5]. It, therefore, always interpolates the *expensive data* while utilizing the *cheap data* as weights which influence the interpolation mainly in the regions where *expensive data* are absent.

The co-Kriging predicts a function value as follows:

$$Z^{*}(\mathbf{x})=\sum_{i=1}^{N_c}\alpha_i(\mathbf{x})Z_{\mathrm{c}}(\mathbf{x}_i)+\sum_{j=1}^{N_e}\beta_j(\mathbf{x})Z_{\mathrm{e}}(\mathbf{x}_j) \tag{1}$$

where N_c and N_e are the numbers of *cheap* and *expensive data*, respectively, $Z_\mathrm{c}(\cdot)$ and $Z_\mathrm{e}(\cdot)$ are objective function values from *cheap* and *expensive data*, respectively, and the weighting coefficients $\alpha_i(\cdot)$ and $\beta_j(\cdot)$ are found from the following equations:

$$\begin{aligned}
&\textstyle\sum_{k=1}^{n}\alpha_k Cov[Z_{\mathrm{c}}(\mathbf{x}_i),Z_{\mathrm{c}}(\mathbf{x}_k)]+\sum_{q=1}^{m}\beta_q Cov[Z_{\mathrm{c}}(\mathbf{x}_i),Z_{\mathrm{e}}(\mathbf{x}_q)]\\
&\qquad+\mu_1=Cov[Z_{\mathrm{c}}(\mathbf{x}_i),Z(\mathbf{x})],\quad i=1,\cdots,n\\
&\textstyle\sum_{k=1}^{n}\alpha_k Cov[Z_{\mathrm{c}}(\mathbf{x}_k),Z_{\mathrm{e}}(\mathbf{x}_j)]+\sum_{q=1}^{m}\beta_q Cov[Z_{e}(\mathbf{x}_q),Z_{\mathrm{e}}(\mathbf{x}_j)]\\
&\qquad+\mu_2=Cov[Z_{\mathrm{e}}(\mathbf{x}_j),Z(\mathbf{x})],\quad j=1,\cdots,m\\
&\textstyle\sum_{k=1}^{n}\alpha_k=1,\quad \sum_{q=1}^{m}\beta_q=0
\end{aligned} \tag{2}$$

where $Cov(\cdot,\cdot)$ is the Gaussian covariance function of which the best correlation parameter θ is found by using maximum likelihood estimation, μ_1 and μ_2 are Lagrange multipliers.

In the numerical implementation of the co-Kriging, the number of *expensive data* is kept very small while that of *cheap data* is allowed to be relatively large.

III. Co-Kriging Assisted PSO Algorithm

A PSO algorithm assisted by the co-Kriging is summarized as follows:

Step 1: Construction of *cheap data* ($\boldsymbol{X}_\mathrm{c}$, $\boldsymbol{Z}_\mathrm{c}$) : generate initial N_c sampling points ($\boldsymbol{X}_\mathrm{c}$) by using Latin hypercube design in the whole design space, and calculate the objective function values ($\boldsymbol{Z}_\mathrm{c}$) for the $\boldsymbol{X}_\mathrm{c}$. In this paper, the circulating current losses are obtained through axisymmetric 2-D non-linear time-stepping FEA.

Step 2: Construction of *expensive data* ($\boldsymbol{X}_e$, $\boldsymbol{Z}_e$) : select N_e sampling points from the $\boldsymbol{X}_c$ to form $\boldsymbol{X}_e$ so that $\boldsymbol{X}_e$ is a subset of $\boldsymbol{X}_c$, and calculate the objective function values ($\boldsymbol{Z}_e$) for the $\boldsymbol{X}_e$. In this paper, the circulating current losses are obtained through 3-D non-linear time-stepping FEA.

Step 3: Construction of co-Kriging based on ordinary Kriging utilizing ($\mathbf{X}_c$, $\boldsymbol{Z}_c$) and ($\mathbf{X}_e$, $\boldsymbol{Z}_e$).

Step 4: Find an optimum design by using PSO algorithm where objective function values will be provided from the co-Kriging.

IV. Numerical Examples

A. Analytic Function

In order to investigate the performance of the co-Kriging, a two-dimensional analytic function is taken as an example as follows:

$$f(x_1,x_2)=\sum_{i=1}^{5} i\cos\big((i+1)x_1+i\big)\cdot\sum_{i=1}^{5} i\cos\big((i+1)x_2+i\big) \quad \text{subject to} \quad x_1,x_2\in[-2,0] \tag{3}$$

The *expensive* and *cheap data* are generated as follows:

$$\begin{aligned} &Z_e(\mathbf{x})=f(\mathbf{x}) \\ &Z_c(\mathbf{x})=f(\mathbf{x})+R_{noise}, \quad 0.05\le R_{noise}\le 0.07 \end{aligned} \tag{4}$$

where R_{noise} is random noise.

In order to evaluate and compare the fitting errors, root mean square error (RMSE) and determination coefficient $\mathbf{R}^2$ are defined as follows:

$$\begin{aligned} &\text{RMSE}=\sqrt{\sum_{i=1}^{NTS}[Z^*(\mathbf{x}_i)-Z(\mathbf{x}_i)]^2 \Big/ NTS} \\ &\mathbf{R}^2=\sum_{i=1}^{NTS}\big(Z^*(\mathbf{x}_i)-\bar{Z}(\mathbf{x}_i)\big)\Big/\sum_{i=1}^{NTS}\big(Z(\mathbf{x}_i)-\bar{Z}(\mathbf{x}_i)\big) \end{aligned} \tag{5}$$

where NTS=40×40 is number of testing points uniformly distributed, $Z^*(\cdot)$ and $Z(\cdot)$ are predicted and true values, respectively. It is reported that the fitting quality increases as the determination coefficient approaches to one [6].

Fig. 1 compares the fitting errors for ordinary Kriging (OK) and co-Kriging, where it is found that the co-Kriging, with the help of ten *expensive data*, has a better fitting accuracy than the OK from only *cheap data* without increasing the computing time too much.

Table I compares the fitting performances, where the co-Kriging has determination coefficient of almost one and finds almost same optimum point with true optimum point when it is combined with PSO algorithm.

B. Design of Power Transformer Winding

The low voltage winding of a power transformer, in general, carries out very large current, and is composed of many helical-type parallel conductors to reduce the eddy current loss in winding. As the number of parallel conductors increases, although the eddy current loss in the winding decreases, there exist higher possibility of increasing circulating current loss unless transposition of the windings are not properly designed.

In this paper, an optimal transposition design is achieved by using the suggested co-Kriging assisted PSO algorithm. The

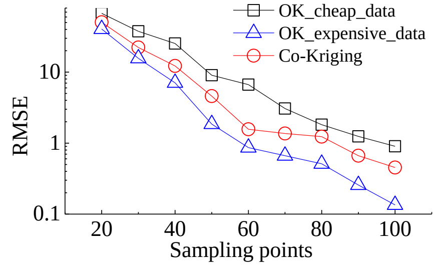

Fig. 1 Fitting errors for analytic function, where sampling data for the OK_cheap_data and OK_expensive_data are composed of only *cheap* and *expensive data*, respectively, and those of co-Kriging include only ten *expensive data*.

TABLE I
Minimization of the Analytic Function

`	N_c	N_e	R^2	Optimal design
Analytical	-	-	-	(-0.8003, -1.4251)
Kriging	100	0	0.9930	(-0.8042, -1.4645)
	0	100	0.999999	(-0.8005, -1.4240)
Co-Kriging	90	10	0.99998	(-0.8005, -1.4229)

TABLE II
Specifications of Power Transformer Model

capacity	Sub-staining bars	High/low voltage	Winding turns	Parallel conductors
370MVA	32	242/20kV	60	92

specifications of a three-phase power transformer model are shown in Table II.

For the construction of sampling data, a few *expensive data* are obtained through 3-D non-linear time-stepping FEA while most sampling data through axisymmetric 2-D time-stepping FEA. The circulating current loss to be minimized is calculated as follows:

$$P_c=\sum_{i=1}^{nc}\left(I_{2i}^2R_{2i}-I_2^2R_2\right) \tag{6}$$

$$\begin{aligned} &\nabla\times\nu\nabla\times\vec{A}=\vec{J} \\ &I_1R_1-e_1=V_1, \quad I_{2i}R_{2i}-e_{2i}=V_2, i=1,2,\cdots,nc \end{aligned} \tag{7}$$

where nc is number of parallel conductors, the subscripts 1 and 2 denote high and low voltage, respectively, and e stands for induced voltage from flux linkage variation.

In the version of full paper, optimal design results will be presented in detail as well as the FEA results.

V. References

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